Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Engineering Mathematics-IV

Subject Code: 4TE04EMT1 **Branch: B.Tech (Civil)**

Semester: 4 **Date: 07/05/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) $curl(grad\ \phi) =$ _______.

(a). 2 (b). 1 (c). 0 (d). -1

b) Let $f(x,y,z) = c$ represent the equation of a surface, Unit normal vector is___

(a). $\frac{\text{grad } f}{|\text{grad } f|}$ (b). grad f (c). div(grad f) (d). curl(grad f)

c) The fixed points of the mapping $w = \frac{-z+1}{z-1}$

(a). 2, 1 (b). 1, -1 (c). 1, -2 (d). 2, -2

d) If $f(z) = u(x,y) + i\,v(x,y)$ is analytic then $f'(z) =$ ______.

(a). $u_x + i\,v_x$ (b). $u_x - i\,v_x$ (c). $u_y + i\,v_x$ (d). $u_x + i\,v_y$

e) The value of $\int_c \frac{dz}{z-10}$. $C: |z| = 1$

(a). $2\pi i$ (b). $-2\pi i$ (c). $4\pi i$ (d). 0

f) if $\bar{A}(t) = 3t^2\hat{\imath} + 2t\hat{\jmath} + 4t^3\hat{k}$, $\int_{t=1}^{t=2} \bar{A}(t)dt\ equal\ to$

(a). $7\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ (b). $7\hat{\imath} + 3\hat{\jmath} + 15\hat{k}$ (c). $-7\hat{\imath} - 3\hat{\jmath} + 15\hat{k}$ (d).None of these

g) In Gauss- elimination method coefficient matrix reduce into

(a). Upper triangular matrix (b). Lower triangular matrix
(c). Unit Matrix (d). Diagonal Matrix

h) Relation between E and Δ

(a). $\Delta = E - 1$ (b). $\Delta = E + 1$ (c). $\Delta = 1 - E^{-1}$ (d). All of these

i) $E^3 f(x) =$ _____
(a). $3f(x+h)$ (b). $f(x+3h)$ (c). f(x − 3h) (d). None of these

j) Putting $n = 1$ in Newton- cote's formulae, we get ______
(a). Trepezoidal Formula (b). Simpson's $\frac{1}{3}$ rule
(c). Simpson's $\frac{1}{3}$ rule (d). None of these

k) Which one of the following method is more rapid in convergence than Gauss-Jacobi method
(a). Gauss- elimination method (b). Gauss- Jordan method
(c). Gauss Seidel method (d). None of these

l) If $f(x)$ is even then
(a). $B(\lambda) = 0$ (b). $A(\lambda) = 0$ (c). Both a and b (d). None of these

m) Write Heat Equation.

n) Write Fourier sine integral formula.

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Given $\vec{u} = xyz\,\hat{\imath} + (2x^2z - y^2x)\,\hat{\jmath} + xz^3\,\hat{k}$ and **(06)**

$v = xy + yz + z^2$ then find $\nabla \cdot \vec{u}$ and $\nabla \cdot v$ and $\nabla \times \vec{u}$.

b) If $\phi(x, y, z) = x^3y + xy^3 + zxy$ then find $\nabla\phi$ and unit normal at (1,1,1). **(04)**

c) Prove that $\vec{f} = \frac{x\,\hat{\imath} + y\,\hat{\jmath}}{x^2 + y^2}$ is solenoidal. **(04)**

Q-3 **Attempt all questions** **(14)**

a) $Evaluate: \oint_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz, where\ C\ is\ the\ circle\ |z| = 3.$ **(05)**

b) $Evaluate\ \oint_c \frac{dz}{z^2 + 9}, where\ \ C$ is $|z - 3i| = 4$. **(05)**

c) Determine the mobius transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -5, w_2 = -1, w_3 = 3$ respectively. What are the invariant points of the transformation? **(04)**

Q-4 **Attempt all questions** **(14)**

a) Using Green's theorem, evaluate $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region bounded by $y^2 = x$ and $y = x^2$. **(05)**

b) Verify Stoke's theorem for $\vec{F} = xy^2\,\hat{\imath} + y\,\hat{\jmath} + z^2x\,\hat{k}$ for the suface of a rectangular lamina bounded by $x = 0, y = 0, x = 1, y = 2, z = 0$. **(05)**

c) Dividing the range into 10 equal parts, find the approximate value of **(04)**

$\int_0^\pi \sin x\, dx$ by using simpson's $\frac{1}{3}$ rule.

Q-5 **Attempt all questions** **(14)**

a) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y \sin 2y)$ **(05)**

b) Solve by using Gauss-Jordan method **(05)**

$2x + y + 4z = 12, \quad 8x - 3y + 2z = 20, \quad 4x + 11y - z = 33$

c) Use Lagrange's interpolation formula to find the value of y when $x = 10$. **(04)**

x	5	6	9	11
y	12	13	14	16

Q-6 **Attempt all questions** **(14)**

a) Sove the following system by using Gauss-Seidel method **(05)**

$27x + 6y - z = 85, \quad 6x + 5y + 2z = 72, \quad x + y + 54z = 110$

b) Using Taylor series method, find $y(1.1)$ correct to four decimal places, given that **(05)**

$$\frac{dy}{dx} = xy^{\frac{1}{3}}, y(1) = 1.$$

c) Obtain Picard's second approximation solution of the initial value problem **(04)**

$\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$ correct to four decimal places, gives that $\mathrm{Y}(0) = 0$

Q-7 **Attempt all questions** **(14)**

a) Given $\sin 45° = 0.7071$,$\sin 50° = 0.7660$,$\sin 55° = 0.8192$, $\sin 60° = 0.8660$ then find $\sin 52°$ using Newton's forward Interpolation formula. **(05)**

b) Find the fourier transform of $e^{-a|x|}, a > 0$ and deduce that **(05)**

$$\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda = \frac{\pi}{2a} e^{-a|x|}$$

c) Find the fourier cosine and sine transforms of the function **(04)**

$$f(x) = \begin{cases} k \ \ if \ \ 0 < x < a \\ 0 \ \ if \ \ \ x > a \end{cases}$$

Q-8 **Attempt all questions** **(14)**

a) Find the fourier transform of $e^{-(a^2x^2)}, a > 0$ and deduce that **(05)**

$$F\left(e^{-\frac{\lambda^2}{2}}\right) = e^{-\frac{\lambda^2}{2}}.$$

b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied. **(05)**

c) If $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 2000$, and $y_4 = 100$ then find $\Delta^4 y_0$. Also write Newton forward interpolation formula. **(04)**